Investigating a Proposed Funding Formula Metric

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Abstract

This presentation describes in detail how mathematical probability is used to investigate the practicability of a proposed metric pertaining to a higher-education funding formula model.

Outline

- Proposed Metric
- Implementation
- Example
- Discussion
- Summary
- Q&A

Proposed Metric

Release the set-aside for next year iff the proportion of this year's graduates who are *successful* exceeds last year's proportion by more than 0.01 (= 1.0%).

A graduate is deemed "successful" iff six months after graduation he/she is either enrolled in grad or prof school, or employed in a position "commensurate" with his bachelor's degree.

Proposed Metric

Release the set-aside for ~~next year~~ year after next iff the proportion of this year's graduates who are *successful* exceeds last year's proportion by more than 0.01 (= 1.0%).

A graduate is deemed "successful" iff six months after graduation he/she is either enrolled in grad or prof school, or employed in a position "commensurate" with his bachelor's degree.

Proposed Metric

N_i No. of year i grads.

G_i No. of year i grads enrolled in grad/prof school.

W_i No. of year i grads working successfully.

$p_i = (G_i + W_i) / N_i$ Pop. proportion of "successful" year i grads.

$p_2 - p_1$ Change in consecutive pop. proportions.

Release the set-aside for year *4* iff $p_2 - p_1 > 0.01$.

Proposed Metric

N_i No. of year i grads. Know

G_i No. of year i grads enrolled in grad/prof school. "Know"

W_i No. of year i grads working successfully. ???

$p_i = (G_i + W_i) / N_i$ Pop. proportion of "successful" year i grads. ???

$p_2 - p_1$ Change in consecutive pop. proportions. ???

Release the set-aside for year 4 iff $p_2 - p_1 > 0.01$. ???

How to Proceed?

$R_i = N_i - G_i$ No. of year i grads *not* accounted for.

Solution #1: Survey all R_i unknown year i grads, and observe W_i .

- Need ~100% response rate (to reduce reporting bias).
- Expensive approach.

How to Proceed?

$R_i = N_i - G_i$ No. of unknown year i grads.

$u_i = W_i / R_i$ Corres. proportion of unknown year i grads.

$W_i = u_i R_i$

Solution #2: Survey a SRS(n_i) of the R_i unknown year i grads, observe the number x_i of successful grads, and *estimate* W_i by estimating u_i using x_i / n_i .

- Need ~100% response rate, but this seems more attainable here.
- Statistical approach which promises to be less expensive.
- Allows one to quantify decision uncertainty.

Solution #2: Probability Results

1. $$\hat{u} \equiv \frac{x}{n} \dot{\sim} Normal\left(u \equiv \frac{W}{R} \equiv \frac{W}{N-G} \;\; , \;\; \sigma_{\hat{u}} \equiv \sqrt{\frac{u(1-u)}{n} \cdot \frac{R-n}{R-1}} \right)$$

2. $$\sigma_{\hat{u}} \equiv \sqrt{\frac{u(1-u)}{n} \cdot \frac{R-n}{R-1}}$$

3. $$p \equiv (G+W)/N = (G+Ru)/N$$

4. $$\hat{p} \equiv (G+R\hat{u})/N$$

Solution #2: Probability Results (cont.)

5. $\hat{p} \dot{\sim} Normal\left(p \equiv \frac{G+Ru}{N} = \frac{G+W}{N} \ , \ \sigma_{\hat{u}} \cdot \left(\frac{R}{N} \right) \right)$

6. $n = \frac{R}{1 + \frac{R-1}{u(1-u)} \left(\frac{me \cdot N}{z_{\alpha/2} R} \right)^2} \leq \frac{R}{1 + (R-1) \left(\frac{me \cdot 2N}{z_{\alpha/2} R} \right)^2}$

7. $\hat{p} \pm z_{\alpha/2} \cdot \sigma_{\hat{u}} \cdot \left(\frac{R}{N} \right)$

Solution #2: Probability Results (cont.)

8. $$\left(\hat{p}_2 - \hat{p}_1\right) \dot{\sim} Normal\left(\left(p_2 - p_1\right)\ ,\ \sqrt{\sigma_{\hat{u}_2}^2 \cdot \left(\frac{R_2}{N_2}\right)^2 + \sigma_{\hat{u}_1}^2 \cdot \left(\frac{R_1}{N_1}\right)^2}\right)$$

9. $$\left(\hat{p}_2 - \hat{p}_1\right) \ \pm\ z_{\alpha/2}\sqrt{\sigma_{\hat{u}_2}^2 \cdot \left(\frac{R_2}{N_2}\right)^2 + \sigma_{\hat{u}_1}^2 \cdot \left(\frac{R_1}{N_1}\right)^2}$$

10. $$Z \equiv \frac{\left(\hat{p}_2 - \hat{p}_1\right) - \left(p_2 - p_1\right)}{\sqrt{\sigma_{\hat{u}_2}^2 \cdot \left(\frac{R_2}{N_2}\right)^2 + \sigma_{\hat{u}_1}^2 \cdot \left(\frac{R_1}{N_1}\right)^2}}$$

Solution #2: Hypothesis Test

Hypotheses:

$$H_0: \; p_2 - p_1 \le 0.01$$

$$H_A: \; p_2 - p_1 > 0.01$$

Decision Rule: Reject the null hypothesis (*i.e.*, release the year 4 set-aside) at the approx. $\alpha = 0.05$ (say) level of significance iff

$$Z \equiv \frac{\left(\hat{p}_2 - \hat{p}_1\right) - 0.01}{\sqrt{\sigma^2_{\hat{u}_2} \cdot \left(\frac{R_2}{N_2}\right)^2 + \sigma^2_{\hat{u}_1} \cdot \left(\frac{R_1}{N_1}\right)^2}} > z_\alpha = z_{0.05} = 1.645$$

Solution #2: Steps

1. Learn the no. of year 1 grads: N_1
2. Learn the no. of year 1 grads in grad/prof school: G_1
3. Determine the sample size for the survey of year 1 grads: n_1
4. Survey SRS(n_1) of the $R_1 \equiv N_1 - G_1$ year 1 grads, …
5. … follow up, *etc.*, …
6. … and compute the estimated proportion of year 1 grads who are "successful": $\hat{p}_1$

7. Do same for year 2 grads: $\hat{p}_2$

8. Test $H_0: \ p_2 - p_1 \leq 0.01$ against $H_A: \ p_2 - p_1 > 0.01.$

Solution #2: Example

1. FY2011 Bachelors, MO 4-year Publics. (See Handout.)

2. FY2012 Bachelors, MO 4-year Publics. (See Handout.)

3. Example (simulated).

Solution #2: Example (cont.)

		FY2011	FY2012	
N	No. graduates	2,092	1,963	
G	No. in grad/prof school	585	549	Assume 28% of N
$R = N - G$	The rest	1,507	1,414	
n	Sample size	1,402	1,321	Target m.e. = 0.005
$u = W / R$				
x	No. successful grads in sample	1,000	975	For example
est(u) = x / n		0.7133	0.7381	
est(p)		0.7934	0.8113	
m.e. for p		0.0045	0.0044	
est($p_2 - p_1$)			0.0179	
m.e. for ($p_2 - p_1$)			0.0063	
Z			2.465*	Release $$$

Solution #2: Remarks

1. Straightforward application of basic mathematical statistics and probability theory.
   - Straightforward implementation of the proposed funding formula metric.
   - Provides, additionally, a statement of uncertainty.

2. Show me success!

3. Practical (*c.f.,* practicable)?

4. Can be altered to address *known* grads *vs. unknown* grads (rather than grads in prof/grad school *vs.* grads not in prof/grad school).

Solution #2: Remarks (cont.)

5. The *real* metric: Release the set-aside for year after next iff this year's 3-year weighted proportion of graduates who are *successful* exceeds last year's 3-year weighted proportion by more than 0.001 (= 0.10%).

$$p_3 \equiv \frac{(G_1 + W_1) + (G_2 + W_2) + (G_3 + W_3)}{N_1 + N_2 + N_3}$$

$$p_4 \equiv \frac{(G_2 + W_2) + (G_3 + W_3) + (G_4 + W_4)}{N_2 + N_3 + N_4}$$

Release the set-aside for **year 6** iff $p_4 - p_3 > 0.001$.

Summary

- Described a (distilled version of a) funding formula metric.
- Motivated and described a solution for implementing this metric, developed from mathematical probability.
- Presented examples.
- Critiqued this solution.

Questions

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	BACHELOR'S								
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PUBLIC BACCALAUREATE AND HIGHER DEGREE-GRANTING INSTITUTIONS	**FY11 Total (N)**	**NSC per-student cost ($/student)**	**NSC Total Cost ($)**	**% Going on to Grad or Prof School**	**No. Going On to Post Bacc (G)**	**No. Potentially Employed (R = N - G)**	**Required increase in p to satisfy Alumni Success Funding Measure**	**Target Margin of Error for p (m.e.)**	**z (corres. to a specified confidence level)**
		0.12		**28%**			**0.0100**		**1.960**
Harris-Stowe State University	141	0.12	17	0.28	39	102	0.0100	0.0050	1.960
Lincoln University	317	0.12	38	0.28	88	229	0.0100	0.0050	1.960
Missouri Southern State University	860	0.12	103	0.28	240	620	0.0100	0.0050	1.960
Missouri State University-Springfield	3,007	0.12	361	0.28	841	2,166	0.0100	0.0050	1.960
Missouri University of S&T	**997**	**0.12**	**120**	**0.28**	**279**	**718**	**0.0100**	**0.0050**	**1.960**
Missouri Western State University	658	0.12	79	0.28	184	474	0.0100	0.0050	1.960
Northwest Missouri State University	963	0.12	116	0.28	269	694	0.0100	0.0050	1.960
Southeast Missouri State University	1,470	0.12	176	0.28	411	1,059	0.0100	0.0050	1.960
Truman State University	1,170	0.12	140	0.28	327	843	0.0100	0.0050	1.960
University of Central Missouri	1,709	0.12	205	0.28	478	1,231	0.0100	0.0050	1.960
University of Missouri-Columbia	**5,087**	**0.12**	**610**	**0.28**	**1,424**	**3,663**	**0.0100**	**0.0050**	**1.960**
University of Missouri-Kansas City	**1,523**	**0.12**	**183**	**0.28**	**426**	**1,097**	**0.0100**	**0.0050**	**1.960**
University of Missouri-St Louis	**2,092**	**0.12**	**251**	**0.28**	**585**	**1,507**	**0.0100**	**0.0050**	**1.960**

.960 for 95% confidence (Usually used when reporting a "margin of error.")
.645 for 90% confidence
.283 for 80% confidence

Best prior guess for u (use 0.5 to be maximally conservative)	Optimal Sample Size (n)	Sampling Fraction	Cost Initial Survey ($)	Total Cost Initial Data Gathering	Response Rate Initial Survey	Size of 1st Follow-up Survey	Cost of 1st Follow-up ($)	Total Cost After 1st Follow-up ($)	Response Rate of 1st Follow-up	Size of 2nd Follow-up
0.50			6		45%		6		60%	
0.50	102	1.0000	612	629	0.45	57	342	971	0.60	23
0.50	227	0.9913	1,362	1,400	0.45	125	750	2,150	0.60	50
0.50	602	0.9710	3,612	3,715	0.45	332	1,992	5,707	0.60	133
0.50	1,954	0.9021	11,724	12,085	0.45	1,075	6,450	18,535	0.60	430
0.50	**694**	**0.9666**	**4,164**	**4,284**	**0.45**	**382**	**2,292**	6,576	**0.60**	153
0.50	464	0.9789	2,784	2,863	0.45	256	1,536	4,399	0.60	103
0.50	671	0.9669	4,026	4,142	0.45	370	2,220	6,362	0.60	148
0.50	1,006	0.9500	6,036	6,212	0.45	554	3,324	9,536	0.60	222
0.50	809	0.9597	4,854	4,994	0.45	445	2,670	7,664	0.60	178
0.50	1,160	0.9423	6,960	7,165	0.45	638	3,828	10,993	0.60	256
0.50	**3,095**	**0.8449**	**18,570**	**19,180**	**0.45**	**1,703**	**10,218**	29,398	**0.60**	682
0.50	**1,040**	**0.9480**	**6,240**	**6,423**	**0.45**	**572**	**3,432**	9,855	**0.60**	229
0.50	**1,402**	**0.9303**	**8,412**	**8,663**	**0.45**	**772**	**4,632**	13,295	**0.60**	309

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		0.12		**28%**			**0.0100**		**1.960**
Harris-Stowe State University	160	0.12	19	0.28	44	116	0.0100	0.0050	1.960
Lincoln University	302	0.12	36	0.28	84	218	0.0100	0.0050	1.960
Missouri Southern State University	865	0.12	104	0.28	242	623	0.0100	0.0050	1.960
Missouri State University-Springfield	3,226	0.12	387	0.28	903	2,323	0.0100	0.0050	1.960
Missouri University of S&T	**1,077**	**0.12**	**129**	**0.28**	**301**	**776**	**0.0100**	**0.0050**	**1.960**
Missouri Western State University	708	0.12	85	0.28	198	510	0.0100	0.0050	1.960
Northwest Missouri State University	1,148	0.12	138	0.28	321	827	0.0100	0.0050	1.960
Southeast Missouri State University	1,652	0.12	198	0.28	462	1,190	0.0100	0.0050	1.960
Truman State University	1,286	0.12	154	0.28	360	926	0.0100	0.0050	1.960
University of Central Missouri	1,874	0.12	225	0.28	524	1,350	0.0100	0.0050	1.960
University of Missouri-Columbia	**5,528**	**0.12**	**663**	**0.28**	**1,547**	**3,981**	**0.0100**	**0.0050**	**1.960**
University of Missouri-Kansas City	**1,749**	**0.12**	**210**	**0.28**	**489**	**1,260**	**0.0100**	**0.0050**	**1.960**
University of Missouri-St Louis	**1,963**	**0.12**	**236**	**0.28**	**549**	**1,414**	**0.0100**	**0.0050**	**1.960**

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0.50			6		45%		6		60%	
0.50	116	1.0000	696	715	0.45	64	384	1,099	0.60	26
0.50	216	0.9908	1,296	1,332	0.45	119	714	2,046	0.60	48
0.50	605	0.9711	3,630	3,734	0.45	333	1,998	5,732	0.60	134
0.50	2,081	0.8958	12,486	12,873	0.45	1,145	6,870	19,743	0.60	458
0.50	747	0.9626	4,482	4,611	0.45	411	2,466	7,077	0.60	165
0.50	498	0.9765	2,988	3,073	0.45	274	1,644	4,717	0.60	110
0.50	795	0.9613	4,770	4,908	0.45	438	2,628	7,536	0.60	176
0.50	1,124	0.9445	6,744	6,942	0.45	619	3,714	10,656	0.60	248
0.50	885	0.9557	5,310	5,464	0.45	487	2,922	8,386	0.60	195
0.50	1,265	0.9370	7,590	7,815	0.45	696	4,176	11,991	0.60	279
0.50	3,319	0.8337	19,914	20,577	0.45	1,826	10,956	31,533	0.60	731
0.50	1,186	0.9413	7,116	7,326	0.45	653	3,918	11,244	0.60	262
0.50	1,321	0.9342	7,926	8,162	0.45	727	4,362	12,524	0.60	291